basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE/
NASIONALE SENIOR SERTIFIKAAT

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

NOVEMBER 2021

MARKING GUIDELINES/*NASIENRIGLYNE*

MARKS/PUNTE: 150

These marking guidelines consist of 24 pages.
Hierdie nasienriglyne bestaan uit 24 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, sien slegs die EERSTE poging na.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, sien die doodgetrekte poging na.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienriglyne toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Om antwoorde/waardes te aanvaar om 'n probleem op te los, word NIE toegelaat NIE.*

GEOMETRY • *MEETKUNDE*	
S	**A mark for a correct statement (A statement mark is independent of a reason)**
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede)
R	**A mark for the correct reason (A reason mark may only be awarded if the statement is correct)**
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is)
S/R	**Award a mark if statement AND reason are both correct**
	Ken 'n punt toe as die bewering EN rede beide korrek is

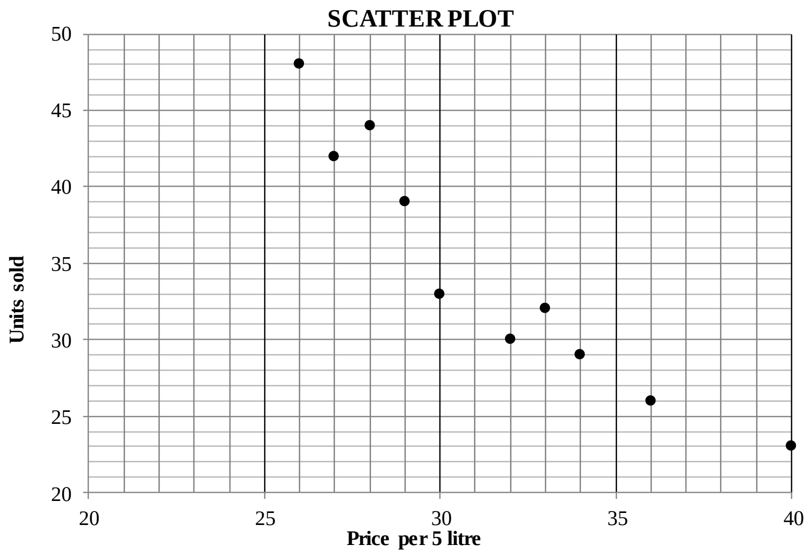
QUESTION/*VRAAG* 1

10	11	13	14	14	15	16	18	18
19	19	20	21	35	35	37	40	41

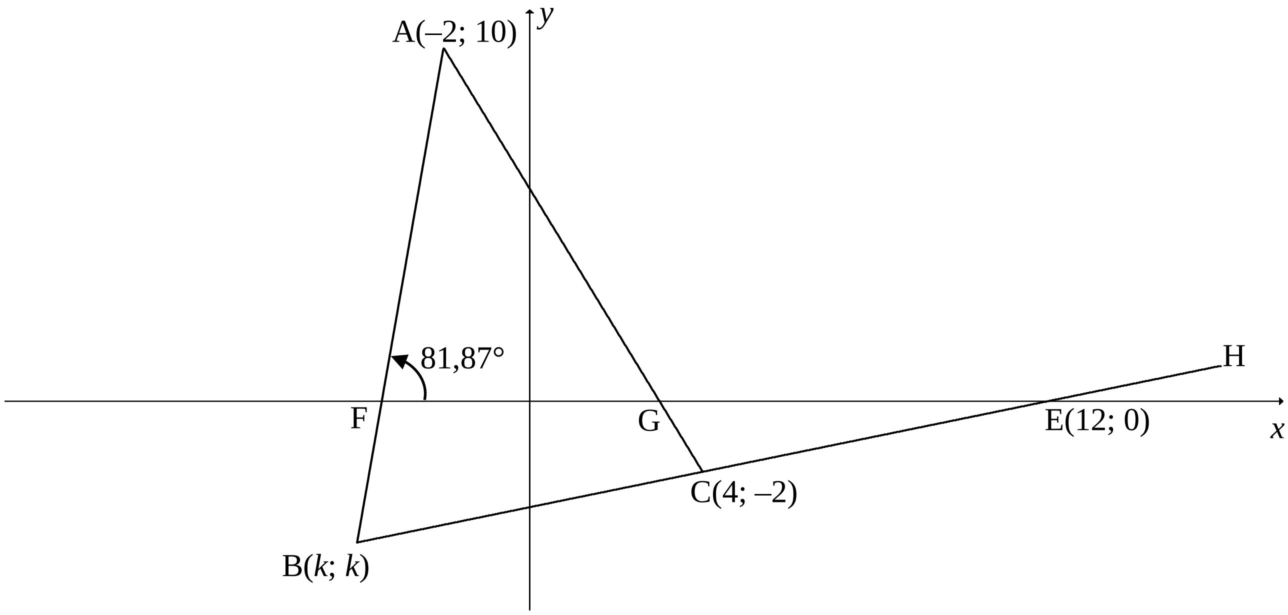
1.1.1	$\bar{x} = \frac{396}{18}$ $\bar{x} = 22$ Answer only: Full marks *Slegs antw: Volpunte*	✓ 396 ✓ answer (2)
1.1.2	σ = 10,1707 ≈ 10,17	✓ answer (1)
1.1.3	$\bar{x} + \sigma = 32{,}17$ $\therefore$ 5 days	✓ 32,17 ✓ 5 (2)
1.2	$22 \times 18 = 396$ ordered/*bestel* $20 \times 18 = 360$ sold/*verkoop* Total not sold/*Totaal nie verkoop nie*: 36 **OR/*OF*** 22–20 = 2 $2 \times 18 = 36$	✓ $18\bar{x}_1$ and $18\bar{x}_2$ ✓ answer (2) ✓ $\bar{x}_1 - \bar{x}_2$ ✓ answer (2)
1.3.1	Option B/*Opsie B* Any one of the following reasons/*Enige een van die vlg redes*: • Median/*Mediaan* = 18,5 • $Q_1 = 14$ • IQR = 21 • Mean > Median, therefore the data is skewed to the right	✓ B ✓ reason (2)
1.3.2	Data is positively skewed/skewed to the right *Data is positief skeef/skeef na regs*	✓ answer (1)
		[10]

QUESTION/*VRAAG* 2

Price of milk in rands per 5-litre container (x) ***Prys van melk in rand, per 5 liter-houer (x)***	26	32	36	28	40	33	29	34	27	30
Number of 5-litre containers of milk sold (y) ***Aantal 5 liter-houers melk verkoop (y)***	48	30	26	44	23	32	39	29	42	33

2.1		1 mark: 3 to 5 points plotted correctly 2 marks: 6 to 9 points plotted correctly 3 marks: all points plotted correctly (3)
2.2	$a = 90{,}478... \approx 90{,}48$ $b = -1{,}773... \approx -1{,}77$ $\hat{y} = 90{,}48 - 1{,}77x$ Answer only: Full marks *Slegs antw: Volpunte*	✓ a ✓ b ✓ equation (3)
2.3	$y = 23{,}069... \approx 23{,}07$ units/*eenhede* (calculator/*sakrekenaar*) **OR/*OF*** $y = 90{,}48 - 1{,}77(38)$ $y = 23{,}22$ units/*eenhede*	✓✓ answer (2) ✓ substitution ✓ answer (2)
2.4	$r = -0{,}94$ The value of r indicates a strong relationship between the cost per 5 litre and the number of units sold $\therefore$ there is a good chance of the prediction being accurate./ *Die waarde van r dui 'n sterk vewantskap tussen die koste per 5 liter en die aantal eenhede verkoop aan $\therefore$ daar is 'n goeie kans dat die voorspelling akkuraat is*	✓ value of r **OR/OF** strong relationship/ *sterk verwantskap* ✓ accurate/*akkuraat* (2)
		[10]

QUESTION/*VRAAG* 3

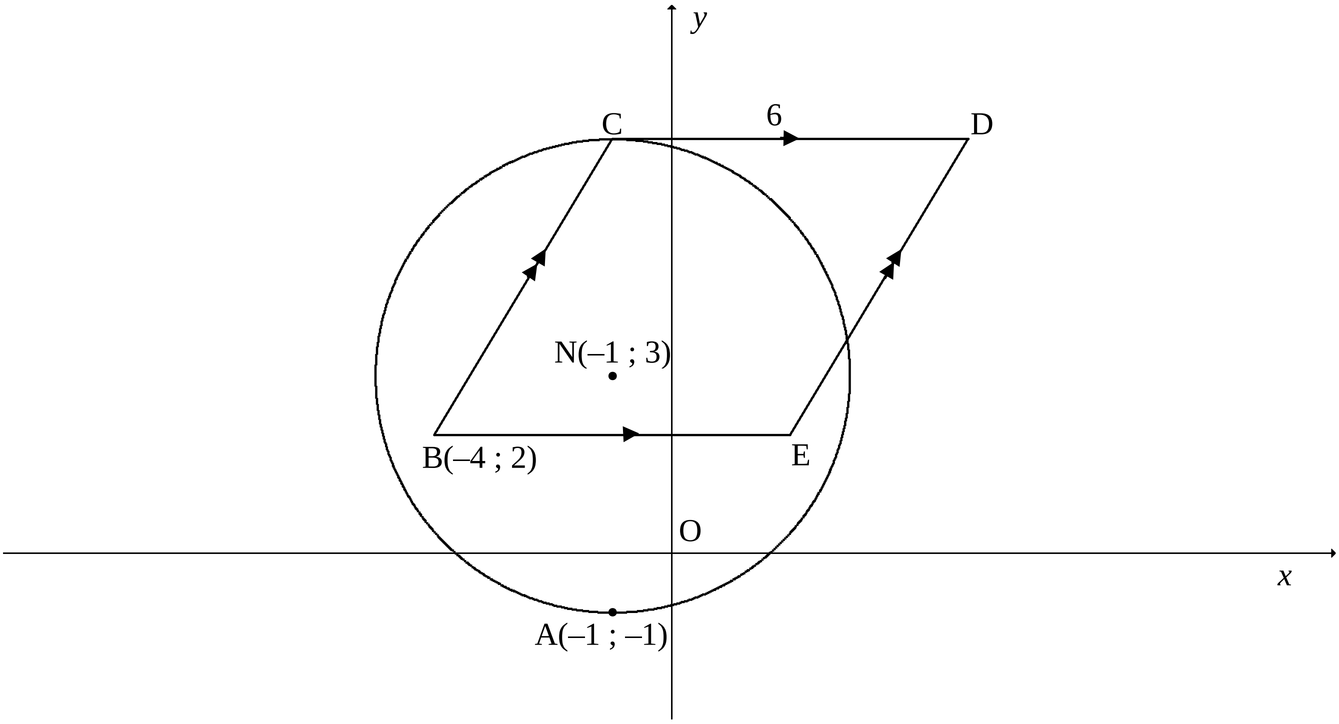


3.1.1	$m_{BE} = m_{CE} = \frac{0-(-2)}{12-4}$ **OR/*OF*** $m_{BE} = m_{CE} = \frac{-2-0}{4-12}$ $= \frac{1}{4}$ $= \frac{1}{4}$	✓ substitution C & E ✓ answer (2)
3.1.2	$m_{AB} = \tan 81{,}87°$ $m_{AB} = 7$ Answer only: Full marks *Slegs antw: Volpunte*	✓ substitution ✓ answer (2)
3.2	$y = mx + c$ or $y - y_1 = m(x - x_1)$ $0 = \frac{1}{4}(12) + c$ or $y - 0 = \frac{1}{4}(x - 12)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$ **OR/*OF*** $y = mx + c$ or $y - y_1 = m(x - x_1)$ $-2 = \frac{1}{4}(4) + c$ or $y - (-2) = \frac{1}{4}(x - 4)$ $c = -3$ $y = \frac{1}{4}x - 3$ $y = \frac{1}{4}x - 3$	✓ substitution of E ✓ answer (2) ✓ substitution of C ✓ answer (2)

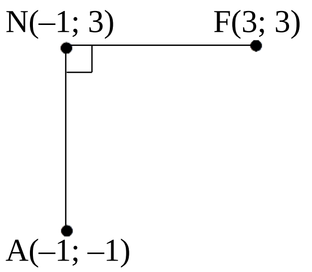
3.3.1	$y=\frac{1}{4}x-3$ $k=\frac{1}{4}k-3$ $\frac{3}{4}k=-3$ $k=-4$ $\therefore$ B(–4; –4)	✓ substitution ✓ answer (2)
	OR/*OF* $m_{BE}=\frac{1}{4}$ **OR/*OF*** $m_{BE}=\frac{1}{4}$ $\frac{0-k}{12-k}=\frac{1}{4}$ $\quad$ $\frac{k}{k-12}=\frac{1}{4}$ $-4k=12-k$ $\quad$ $4k=k-12$ $k=-4$ $\quad$ $k=-4$ $\therefore$ B(–4; –4)	✓ substitution ✓ answer (2)
	OR/*OF* $m_{AB}=\tan 81{,}87°$ $m_{AB}=7$ $m_{AB}=\frac{10-k}{-2-k}$ $7(-2-k)=10-k$ $-14-7k=10-k$ $-6k=24$ $k=-4$ $\therefore$ B(–4; –4)	✓ substitution ✓ answer (2)
	OR/*OF* EB: $y=\frac{1}{4}x-3$ and AB: $y=7x+24$ $\frac{1}{4}x-3=7x+24$ $\frac{27}{4}x=-27$ $x=k=-4$ $\therefore$ B(–4; –4)	✓ equating EB & AB ✓ answer (2)

3.3.2	In ΔAFG: $m_{AC} = \frac{10-(-2)}{-2-4} = -2$ $\tan\theta = m_{AC} = -2$ $\theta = 180° - 63{,}43...°$ $\therefore \theta = 116{,}57°$ $\therefore \hat{A} = 116{,}57° - 81{,}87°$ [ext $\angle$ of Δ] $\therefore \hat{A} = 34{,}70°$ **OR/*OF*** In ΔABC: $a = BC = 2\sqrt{17}; b = AC = 6\sqrt{5}; c = AB = 10\sqrt{2}$ $a^2 = b^2 + c^2 - 2bc.\cos A$ $\left(2\sqrt{17}\right)^2 = \left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - 2(6\sqrt{5})(10\sqrt{2}).\cos A$ $\cos A = \frac{\left(6\sqrt{5}\right)^2 + \left(10\sqrt{2}\right)^2 - \left(2\sqrt{17}\right)^2}{2\left(6\sqrt{5}\right)\left(10\sqrt{2}\right)}$ $= 0{,}822...$ $\therefore A = 34{,}7°$	✓ $m_{AC} = -2$ ✓ $\tan\theta = -2$ ✓ $\theta = 116{,}57°$ ✓ answer (4) ✓ all 3 lengths ✓ substitution into the correct cosine rule ✓ cos A subject ✓ answer (4)
3.3.3	$M\left(\frac{12+(-2)}{2}; \frac{10+(0)}{2}\right)$ Diagonals intersect at the point (5 ; 5)	✓ *x*-value ✓ *y*-value (2)
3.4.1	BE = ET $4\sqrt{17} = \sqrt{(12-p)^2 + (0-p)^2}$ $\left(4\sqrt{17}\right)^2 = \left(\sqrt{(12-p)^2 + (0-p)^2}\right)^2$ $272 = 144 - 24p + p^2 + p^2$ $p^2 - 12p - 64 = 0$ $(p-16)(p+4) = 0$ $\therefore p = 16$ or $p = -4$ (n.a.) $\therefore$ T(16; 16)	✓ substitution of E & T ✓ equating ✓ standard form ✓ factors ✓ $p = 16$ (5)
3.4.2a	$(x-12)^2 + y^2 = \left(4\sqrt{17}\right)^2 = 272$	✓ LHS ✓ RHS (2)
3.4.2b	$m_{radius} = \frac{1}{4}$ $m_{tangent} = -4$ $y = -4x + c$ **OR/*OF*** $y - y_1 = -4(x - x_1)$ $-4 = -4(-4) + c$ $y-(-4) = -4(x-(-4))$ $c = -20$ $y = -4x - 20$ $y = -4x - 20$	✓ $m_{tangent}$ ✓ substitution of B ✓ equation (3)
		[24]

QUESTION/*VRAAG* 4



4.1	Radius = 4 units/*eenhede*	✓ answer (1)
4.2.1	$CD \perp CN$ $\therefore C(-1\ ;\ 7)$	✓x value ✓ y value (2)
4.2.2	CD = 6 units $\therefore D(5\ ;\ 7)$	✓x value ✓ y value (2)
4.2.3	$\perp h = 5\,\text{units}$ DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}(6)(5)$ $= 15\,\text{units}^2$ **OR/*OF*** $\perp h = 5\,\text{units}$ DC = 6 units $\text{Area } \Delta BCD = \frac{1}{2}[\text{Area of } \|^m]$ $= \frac{1}{2}[(5)(6)]$ $= 15\,\text{units}^2$	✓ $\perp h = 5\,\text{units}$ ✓ substitution into Area formula ✓ answer (3) ✓ $\perp h = 5\,\text{units}$ ✓ substitution into Area formula ✓ answer (3)

	OR/*OF* Let angle of inclination of BC = α $\tan\alpha = \frac{5}{3}$ $\alpha = 59{,}036...°$ $B\hat{C}D = 180° - \alpha$ $B\hat{C}D = 180° - 59{,}036...°$ $B\hat{C}D = 120{,}96°$ Area $\Delta BCD = \frac{1}{2}\left(\sqrt{34}\right)(6)\sin 120{,}96°$ $= 15$ units^2	✓ $B\hat{C}D = 120{,}96°$ ✓ substitution into Area rule ✓ answer (3)
4.3.1	M(3 ; –1) [reflection of N(–1 ; 3) about the line $y = x$] $\therefore MN = \sqrt{(3-(-1))^2 + (-1-3)^2}$ $MN = \sqrt{32} = 4\sqrt{2} = 5{,}66$ units	✓ coordinates of M (A) ✓ substitution of M&N ✓ answer (3)
4.3.2	M(3 ; –1) $m_{MN} = \frac{3-(-1)}{-1-3} = -1$ MN: $-1 = -(3) + c$ or $y - 3 = -1(x+1)$ $c = 2$ $y - 3 = -x - 1$ $\therefore y = -x + 2$ $y = -x + 2$ $x = -x + 2$ $2x = 2$ $x = 1$ $\therefore y = 1$ midpoint (1 ; 1) **OR/*OF***  N(–1 ; 3) $y_F = y_N = 3$ Reflected about $y = x$ $\therefore$ F(3 ; 3) midpoint $\left(\frac{-1+3}{2}; \frac{-1+3}{2}\right) = (1 ; 1)$	✓ equation of MN ✓ equating AF & MN ✓x value ✓ y value (4) ✓✓ coordinates of F ✓x value ✓ y value (4)

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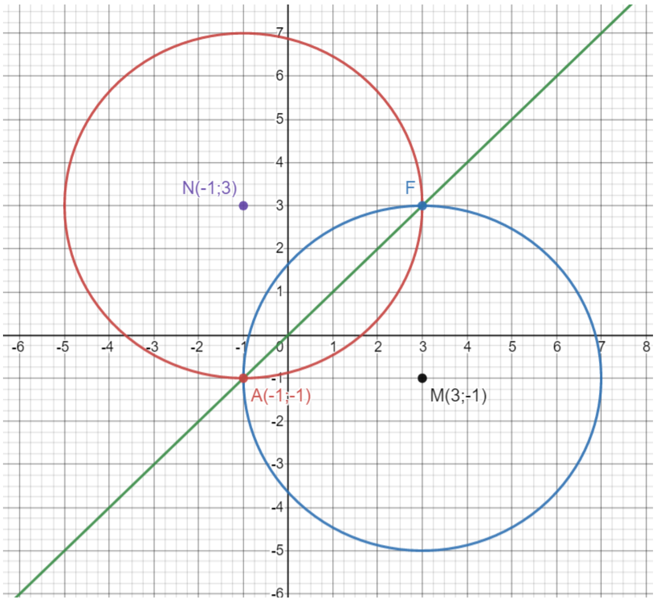
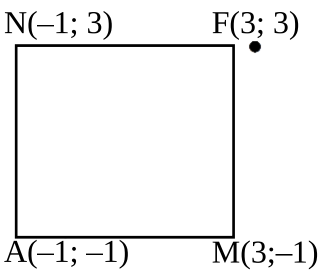
OR/*OF*





NAMF is a square (NA=NF=AM=MF and NA $\perp$ AM)

Midpoint NM $= (1\,;1)$

$= $ Midpoint of AF

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✓ NAMF = square

✓x ✓ y of midpt NM
✓ midpt AF

(4)

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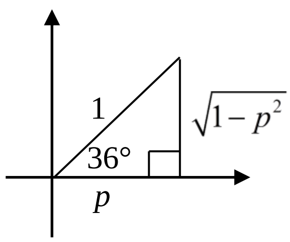
QUESTION/*VRAAG* 5

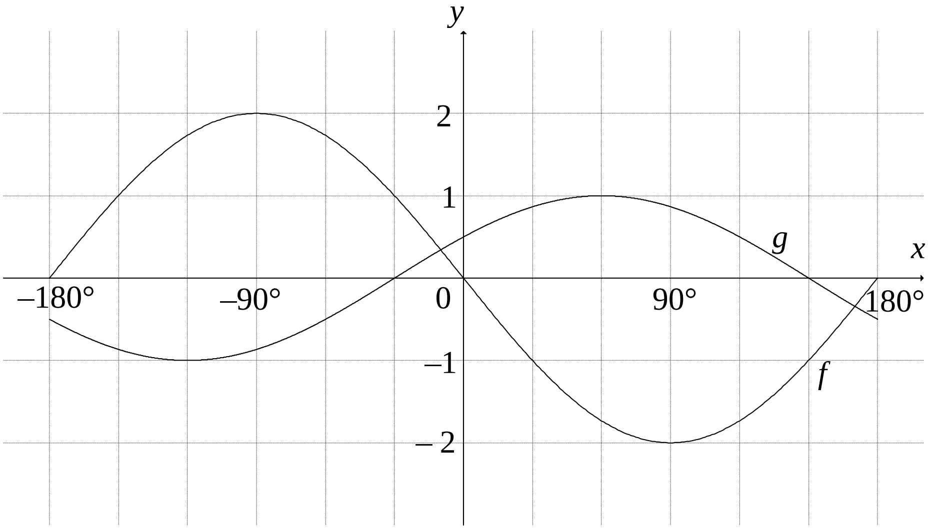
5.1	$\dfrac{\sin 140°.\sin(360°-x)}{\cos 50°.\tan(-x)}$ $=\dfrac{\sin 40°(-\sin x)}{\sin 40°(-\tan x)}$ $=\dfrac{-\sin x}{-\dfrac{\sin x}{\cos x}}$ $=\cos x$	✓ sin40° ✓ $-\sin x$ ✓ co-ratio ✓ $-\tan x$ ✓ $\tan x=\dfrac{\sin x}{\cos x}$ ✓ answer (6)
5.2	$\text{LHS}=\dfrac{-2\sin^2 x+\cos x+1}{1-\cos(540°-x)}$ $\text{RHS}=2\cos x-1$ $\text{LHS}=\dfrac{-2(1-\cos^2 x)+\cos x+1}{1-(-\cos x)}$ $\text{LHS}=\dfrac{-2+2\cos^2 x+\cos x+1}{1+\cos x}$ $\text{LHS}=\dfrac{2\cos^2 x+\cos x-1}{1+\cos x}$ $\text{LHS}=\dfrac{(2\cos x-1)(\cos x+1)}{1+\cos x}$ $\text{LHS}=2\cos x-1$ $\therefore$ LHS = RHS	✓ identity i. t. o. $\cos x$ ✓ $\cos(540°-x)=-\cos x$ ✓ standard form ✓ factors (4)
5.3.1	$\sin 36°=\sqrt{1-p^2}$  $\tan 36°=\dfrac{\sqrt{1-p^2}}{p}$ **OR/*OF*** $\cos^2 36°=1-\sin^2 36°$ $\cos 36°=\sqrt{1-(1-p^2)}$ $=p$ $\tan 36°=\dfrac{\sin 36°}{\cos 36°}$ $=\dfrac{\sqrt{1-p^2}}{p}$	✓ method ✓ value of p ✓ answer (3) ✓ method ✓ cos 36° = p ✓ answer (3)

5.3.2	$\cos 108°$ $= -\cos 72°$ $= -\cos(2 \times 36°)$ $= -(2\cos^2 36° - 1)$ $= -2p^2 + 1$ **OR/*OF*** $\cos 108°$ $= -\cos 72°$ $= -\cos(2 \times 36°)$ $= -(1 - 2\sin^2 36°)$ $= -1 + 2\left(\sqrt{1-p^2}\right)^2$ $= -1 + 2(1 - p^2)$ $= -2p^2 + 1$ **OR/*OF*** $\cos 108°$ $= -\cos 72°$ $= -\cos(2 \times 36°)$ $= -(\cos^2 36° - \sin^2 36°)$ $= -\left(p^2 - \left(\sqrt{1-p^2}\right)^2\right)$ $= -\left(p^2 - (1 - p^2)\right)$ $= -2p^2 + 1$ **OR/*OF*** $\cos 108°$ $= \cos(2 \times 54°)$ $= 2\cos^2 54° - 1$ $= 2(1 - p^2) - 1$ $= 1 - 2p^2$ **OR/*OF*** $\cos 108° = \cos(72° + 36°)$ $= \cos 72° \cos 36° - \sin 72° \sin 36°$ $= (2\cos^2 36° - 1)\cos 36° - (2\sin 36° \cos 36°)\sin 36°$ $= 2\cos^3 36° - \cos 36° - 2\cos 36° \sin^2 36°$ $= 2p^3 - p - 2p\left(\sqrt{1-p^2}\right)^2$ $= 2p^3 - p - 2p + 2p^3$ $= 4p^3 - 3p$	✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) ✓ reduction ✓ double angle ✓ expansion ✓ answer i. t. o. p (4) ✓ double angle ✓✓ expansion ✓ answer i. t. o. p (4) ✓ expansion ✓ both double angle identities ✓ value of sin 36° ✓ answer i. t. o. p (4)
		[17]

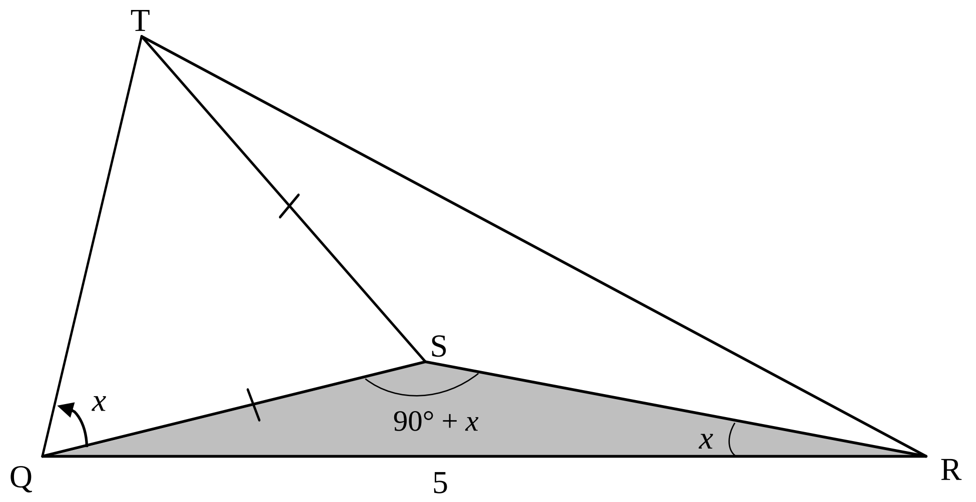
QUESTION/*VRAAG* 6

6.1.1	$\cos(\alpha+\beta)$ $=\cos(\alpha-(-\beta))$ $=\cos\alpha\cos(-\beta)+\sin\alpha\sin(-\beta)$ $=\cos\alpha\cos\beta+\sin\alpha(-\sin\beta)$ $=\cos\alpha\cos\beta-\sin\alpha\sin\beta$	✓ $\cos(\alpha-(-\beta))$ ✓ expansion ✓ reduction (3)
6.1.2	$2\cos 6x\cos 4x-\cos 10x+2\sin^2 x$ $=2\cos 6x\cos 4x-\cos(6x+4x)+2\sin^2 x$ $=2\cos 6x\cos 4x-(\cos 6x\cos 4x-\sin 6x\sin 4x)+2\sin^2 x$ $=\cos 6x\cos 4x+\sin 6x\sin 4x+2\sin^2 x$ $=\cos 2x+2\sin^2 x$ $=1-2\sin^2 x+2\sin^2 x$ $=1$	✓ $\cos 10x=\cos(6x+4x)$ ✓ expansion of $\cos(6x+4x)$ ✓ $\cos 2x$ ✓ $1-2\sin^2 x$ ✓ answer (5)
6.2	$\tan x=2\sin 2x$ $\frac{\sin x}{\cos x}=2(2\sin x\cos x)$ $\sin x=4\sin x\cos^2 x$ $4\sin x\cos^2 x-\sin x=0$ $\sin x(4\cos^2 x-1)=0$ $\sin x=0$ or $\cos^2 x=\frac{1}{4}$ $\cos x=-\frac{1}{2}$ $x=180°+k.360°;\ k\in Z$ or $x=120°+k.360°;\ k\in Z$ $x=240°+k.360°;\ k\in Z$ **OR/*OF*** $\tan x=2\sin 2x$ $\frac{\sin x}{\cos x}=4\sin x\cos x$ $\sin x=4\sin x\cos^2 x$ $4\sin x\cos^2 x-\sin x=0$ $4\sin x(1-\sin^2 x)-\sin x=0$ $3\sin x-4\sin^3 x=0$ $\sin x(3-4\sin^2 x)=0$ $\sin x=0$ or $\sin^2 x=\frac{3}{4}$ $\sin x=\frac{\sqrt{3}}{2}$ or $\sin x=-\frac{\sqrt{3}}{2}$ $x=180^\circ+k.360^\circ,k\in Z$ or $x=120^\circ+k.360^\circ,k\in Z$ or $x=240^\circ+k.360^\circ,k\in Z$	✓ quotient identity ✓ double angle identity ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=120°$& $240°$ **OR/OF** $x=\pm120°$ ✓ $k.360°;\ k\in Z$ (7) ✓ quotient identity ✓ identity ✓ factors ✓ both equations ✓ $x=180°$ ✓ $x=120°$& $240°$ **OR/OF** $x=\pm120°$ ✓ $k.360°;\ k\in Z$ (7)
		[15]

QUESTION/*VRAAG* 7

7.1		✓ both turning points ✓ both x intercepts (–30° & 150°) ✓ shape (3)
7.2	Period = 120°	✓✓ answer (2)
7.3	$x = -30°$	✓ answer (1)
7.4	Range of/*waardeversameling van g*: $y \in [-1\,;1]$ Range of/*Waardeversameling van* $\frac{1}{2}g$: $y \in \left[-\frac{1}{2}\,;\frac{1}{2}\right]$ Range of/*Waardeversameling van* $\frac{1}{2}g + 1$: $y \in \left[\frac{1}{2}\,;\frac{3}{2}\right]$ **OR/*OF*** Range of/*Waardeversameling van* $\frac{1}{2}g + 1$: $\frac{1}{2} \le y \le \frac{3}{2}$	✓ critical values ✓ correct notation (2) ✓ critical values ✓ correct notation (2)
		[8]

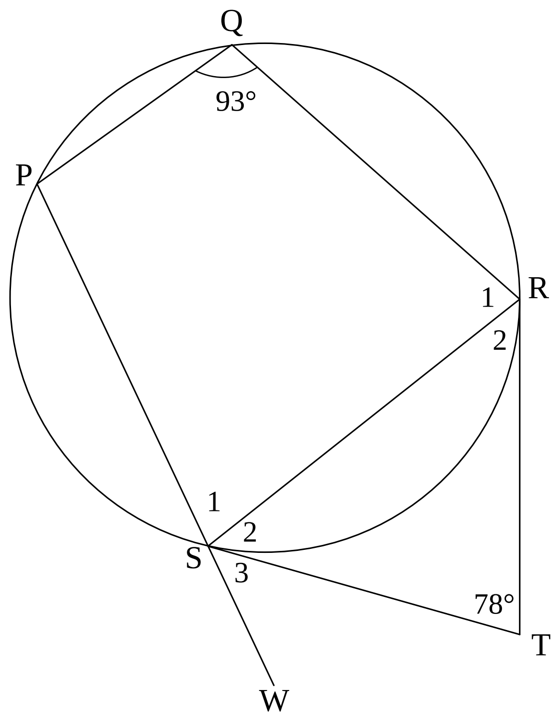
QUESTION/*VRAAG* 8



8.1	In ΔSQR: $\frac{QS}{\sin x} = \frac{QR}{\sin(90° + x)}$ $\frac{QS}{\sin x} = \frac{5}{\cos x}$ $QS = \frac{5\sin x}{\cos x}$ $QS = 5\tan x$	✓ correct use of sine rule ✓ $\sin(90° + x) = \cos x$ ✓ $QS = \frac{5\sin x}{\cos x}$ (3)
8.2	$\frac{QT}{\sin(180° - 2x)} = \frac{TS}{\sin x}$ $\frac{QT}{\sin 2x} = \frac{5\tan x}{\sin x}$ $QT = \frac{5\tan x \sin 2x}{\sin x}$ $QT = \frac{5\left(\frac{\sin x}{\cos x}\right)(2\sin x \cos x)}{\sin x}$ $QT = \frac{5\sin x\,(2\sin x)}{\sin x}$ $QT = 10\sin x$	✓ correct use of sine rule ✓ TS = QS = 5tan$x$ ✓ $QT = \frac{5\tan x \sin 2x}{\sin x}$ ✓ $\tan x = \frac{\sin x}{\cos x}$ ✓ $\sin 2x = 2\sin x \cos x$ (5)

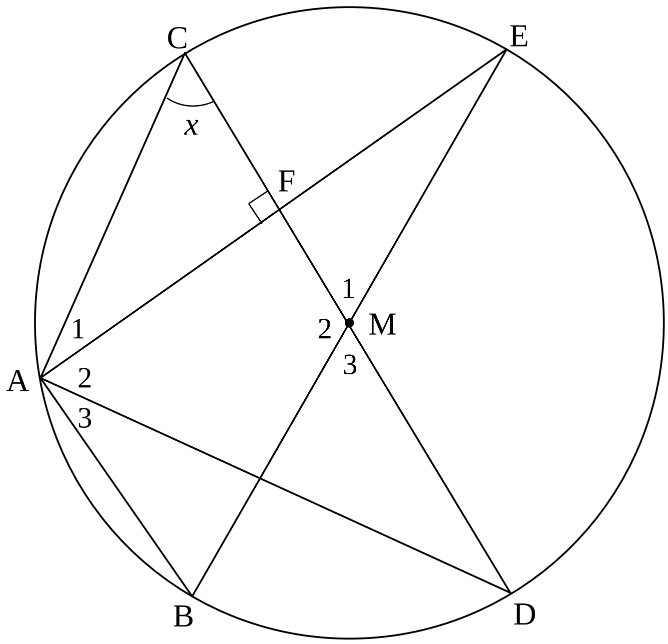
	OR/*OF* $QT^2 = QS^2 + TS^2 - 2QS.TS\cos Q\hat{S}T$ $QT^2 = (5\tan x)^2 + (5\tan x)^2 - 2(5\tan x).(5\tan x)\cos(180° - 2x)$ $QT^2 = 50\tan^2 x - 50\tan^2 x(-\cos 2x)$ $QT^2 = 50\tan^2 x(1+\cos 2x)$ $QT^2 = 50\tan^2 x(1+2\cos^2 x - 1)$ $QT^2 = 50\tan^2 x(2\cos^2 x)$ $QT^2 = 100\dfrac{\sin^2 x}{\cos^2 x}(\cos^2 x)$ $QT^2 = 100\sin^2 x$ $QT = 10\sin x$	✓ correct use of cos rule ✓ TS = QS = 5tan$x$ ✓ $\cos 2x = 2\cos^2 x - 1$ & reduction ✓ $\tan x = \dfrac{\sin x}{\cos x}$ ✓ $QT^2 = 100\sin^2 x$ (5)
	OR/*OF* $TS^2 = QS^2 + TQ^2 - 2QS.TQ.\cos x$ $(5\tan x)^2 = (5\tan x)^2 + TQ^2 - 2(5\tan x).TQ.\cos x$ $0 = TQ^2 - 2(5\tan x).TQ.\cos x$ $0 = TQ[TQ - 10\tan x.\cos x]$ $TQ = 10\tan x.\cos x \quad (TQ \neq 0)$ $= 10\dfrac{\sin x}{\cos x}.\cos x$ $= 10\sin x$	✓ correct use of cos rule ✓ TS = QS = 5tan$x$ ✓ quadratic equation ito TQ ✓ TQ = 10tan x . cos x ✓ $\tan x = \dfrac{\sin x}{\cos x}$ (5)
8.3	Area of $\Delta TQR = \dfrac{1}{2}.TQ.QR\sin T\hat{Q}R$ $= \dfrac{1}{2}(10\sin 25°)(5)(\sin 70°)$ $= 9{,}93\,\text{unit}^2$	✓ correct substitution into the area rule ✓ answer (2)
		[10]

QUESTION/VRAAG 9



9.1	tangents from same(common) point/*raaklyne vanaf dieselfde punt*	✓ R (1)
9.2.1	$\hat{S}_2 = S\hat{R}T$ [$\angle$s opp equal sides/$\angle$*e teenoor gelyke sye*] $\therefore \hat{S}_2 = 51°$ [sum of $\angle$s in Δ/*som van* $\angle$*e in* Δ]	✓ R ✓ S (2)
9.2.2	$\hat{S}_2 + \hat{S}_3 = 93°$ [ext $\angle$ of cyclic quad/*buite*$\angle$ *van koordevh*] $\hat{S}_3 = 42°$ **OR/*OF*** $\hat{S}_1 = 87°$ [opp $\angle$s of cyclic quad/*teenoorst* $\angle$*e v kdvh*] $\hat{S}_3 = 180° - (87° + 51°)$ $\hat{S}_3 = 42°$ [$\angle$s on a str line/$\angle$*e op reguitlyn*]	✓ R ✓ answer (2) ✓ R ✓ answer (2)
		[5]

QUESTION/*VRAAG* 10

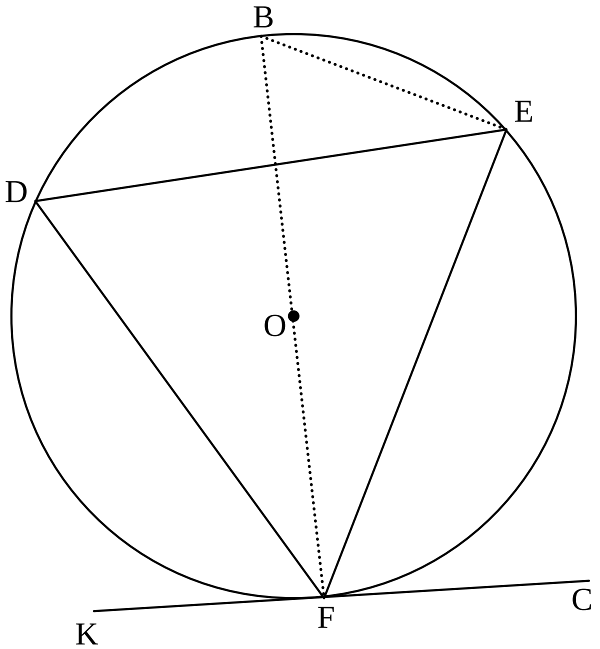


10.1	line from centre ⊥ to chord/*lyn vanaf middelpunt ⊥ op koord*	✓ R (1)
10.2	$\therefore \hat{A}_1 = 90° - x$ [sum of ∠s in Δ/*som van ∠e in Δ*] $\therefore \hat{M}_1 = 180° - 2x$ [∠at centre=2× at circumf/*midpts*∠= 2×omtreks∠]	✓ S ✓ S ✓ R (3)
10.3	$C\hat{A}D = 90°$ [∠ in semi circle/∠ *in halfsirkel*] $\hat{A}_2 = 90° - (90° - x)$ $\hat{A}_2 = x$ $\therefore \hat{A}_2 = \hat{C} = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $E\hat{M}D = 2x$ [adj suppl ∠s/*aanligg suppl ∠e*] $\therefore \hat{A}_2 = x$ [∠at centre= 2×∠ at circumf/*midpts*∠= 2 ×omtreks∠] $\therefore \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $\hat{M}_3 = 180° - 2x$ [vert. opp/ *regoorstaande ∠e*] $\therefore \hat{A}_3 = 90° - x$ [∠at centre= 2×∠ at circumf/*midpts*∠= 2 ×omtreks∠] $B\hat{A}E = 90°$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_2 = C = x$ ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF***	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ S ✓ R ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)

	CD ∥ AB [midpt. Thm/ *middelpuntst.*] $B\hat{A}E = 90^\circ$ [∠ in semi-circle/∠ *in halfsirkel*] $\therefore \hat{A}_3 = \hat{D} = 90^\circ - x$ [alt.∠s; CD∥AB/*verwiss ∠e*] $\therefore \hat{A}_2 = x =$ C ∴ AD is a tangent [converse tan-chord theorem/*omgek rkl-kd st.*] **OR/*OF*** $C\hat{A}D = 90^\circ$ [∠ in semi circle/∠ *in halfsirkel*] AC = diameter [converse ∠ in semi circle/*omgek ∠ in halfsirkel*] ∴ AD is a tangent [converse radius ⊥ tangent/*omgek radius⊥rkl*]	✓ S ✓ R ✓ S ✓ R (4) ✓ S ✓ R ✓ S ✓ R (4)
10.4	AF = FE and BM = ME [given & radii] ∴ FM = $\frac{1}{2}$ AB = 12 units [Midpt Theorem/*middelptstelling*] EM = MB = CM = 18 units [radii] ∴ EB = 36 units [diameter = 2 radius] $\therefore AE^2 = (36)^2 - (24)^2$ [Pythagoras] AE = $12\sqrt{5}$ or 26,83 units **OR/*OF*** AF = FE and BM = ME [given & radii] ∴ FM = $\frac{1}{2}$ AB = 12 units [Midpt Theorem/*middelpuntstelling*] EM = MB = CM = 18 units [radii] $\therefore FE^2 = (18)^2 - (12)^2$ [Pythagoras] FE = $6\sqrt{5}$ AE = $12\sqrt{5}$ or 26,83 units	✓ FM = 12 ✓ R ✓ EB = 36 ✓ using Pyth correctly ✓ answer (5) ✓ FM = 12 ✓ R ✓ EM = 18 ✓ using Pyth correctly ✓ answer (5)
		[13]

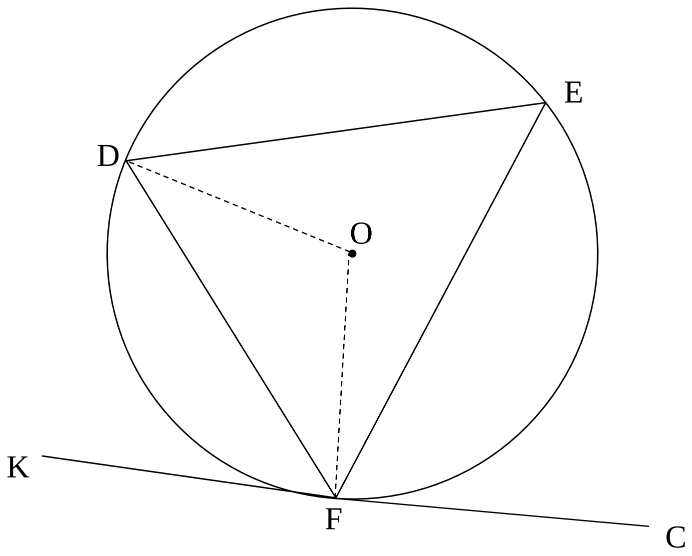
QUESTION/*VRAAG* 11

11.1



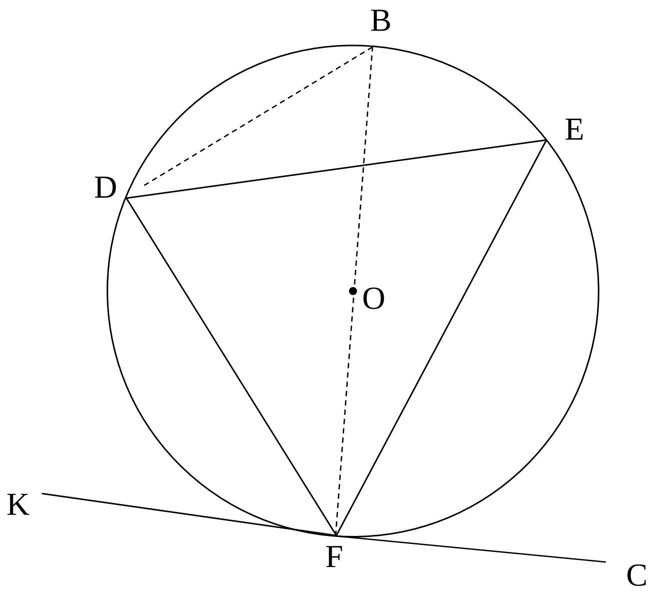
	Construction: Draw diameter BF and draw BE *Konstruksie: Trek middellyn* BF *en verbind* BE	✓Constr
	$B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius ⊥ tangent/*raaklyn*]	✓ S ✓ R
	$B\hat{E}F = 90°$ [∠ in semi-circle/*semi-sirkel*]	✓ S
	$\therefore D\hat{E}F = 90° - B\hat{E}D$	
	$= 90° - B\hat{F}D$ [∠s same segment/∠*e dieselfde segment*]	✓ S/R
	$\therefore D\hat{F}K = D\hat{E}F$	(5)

OR/*OF*



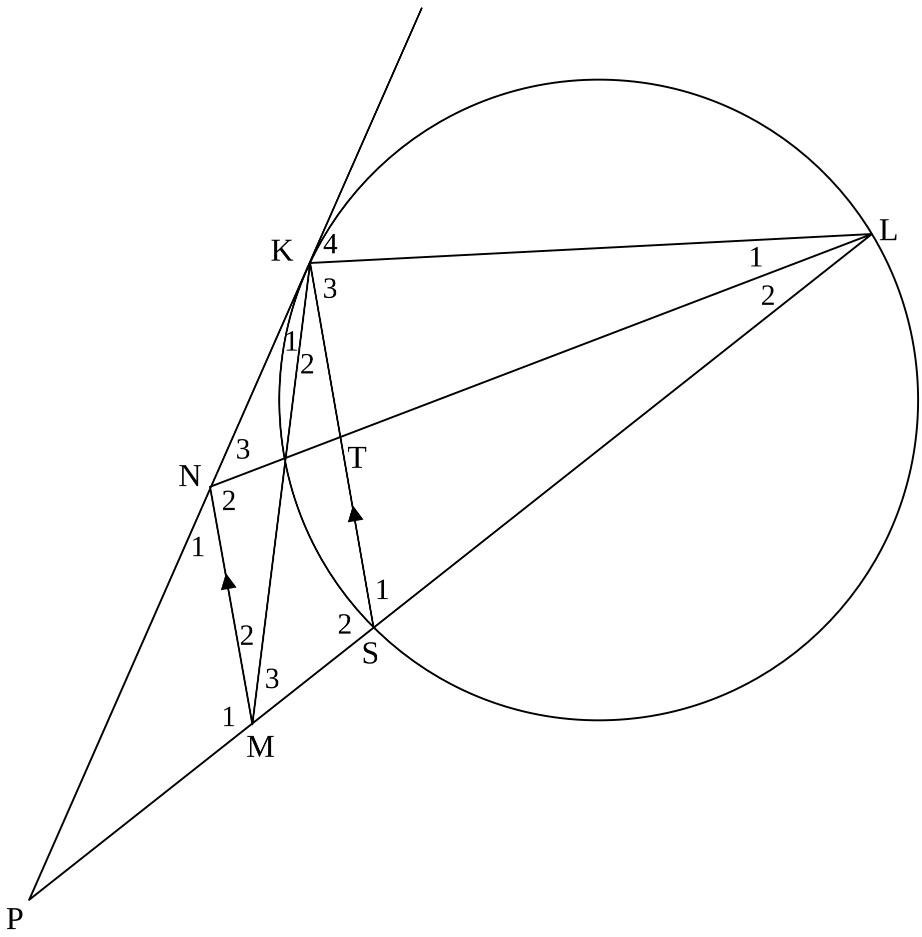
	Construction: Draw radii DO and OF *Konstruksie: Trek radii DO en OF* $\text{O}\hat{\text{F}}\text{K} = 90°$ or $\text{D}\hat{\text{F}}\text{K} = 90° - \text{O}\hat{\text{F}}\text{D}$ radius⊥ tangent/*raaklyn*] $\text{O}\hat{\text{D}}\text{F} = \text{O}\hat{\text{F}}\text{D}$ [∠s opp = sides/∠*e teenoor = sye*] $\therefore \text{D}\hat{\text{O}}\text{F} = 180° - 2\text{O}\hat{\text{F}}\text{D}$ [∠s of Δ/∠*e van Δ*] $\text{D}\hat{\text{E}}\text{F} = 90° - \text{O}\hat{\text{F}}\text{D}$ [∠ at centre = 2×∠ circumf/ *midpts∠ = 2× omtreks∠*] $\therefore \text{D}\hat{\text{F}}\text{K} = \text{D}\hat{\text{E}}\text{F}$	✓ construction ✓ S ✓R ✓ S ✓ S/R (5)

OR/OF



	Construction: Draw diameter BF and join BD. *Konstruksie: Trek middellyn BF en verbind BD.* $B\hat{F}K = 90°$ or $D\hat{F}K = 90° - B\hat{F}D$ [radius⊥ tangent/*raaklyn*] $F\hat{D}B = 90°$ [∠ in half circle/*semi-sirkel*] $\hat{B} = 90° - B\hat{F}D$ $\therefore D\hat{F}K = \hat{B}$ but $\hat{B} = \hat{E}$ [∠s same segment/∠*e dieselfde segment*] $\therefore D\hat{F}K = \hat{E}$	✓ construction ✓ S ✓/R ✓ S ✓ S/R (5)

11.2



11.2.1(a)	$\hat{K}_4 = \hat{S}_1$ [tan chord theorem/*raaklynkoordstelling*] $\hat{M}_2 + \hat{M}_3 = \hat{S}_1$ [corresp ∠s; / *ooreenk ∠s;* MN ‖ KS] $\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$	✓ S ✓ R ✓ S ✓ R (4)
11.2.1(b)	$\therefore \hat{K}_4 = \hat{M}_2 + \hat{M}_3 = N\hat{M}L$ $\therefore$ KLMN is a cyclic quad [ext ∠ of quad = opp int ∠ / *buite ∠ van vh = teenoorst binne ∠*] **OR/OF** $N_1 = \hat{K}_1 + \hat{K}_2 = N\hat{K}S$ [corresp ∠s; / *ooreenk ∠s;* MN ‖ KS] $N\hat{K}S = K\hat{L}S$ [tan chord theorem / *raaklynkoordstelling*] $\hat{N}_1 = K\hat{L}S$ $\therefore$ KLMN is a cyclic quad [ext ∠ of quad = opp int ∠ / *buite ∠ van vh = teenoorst binne ∠*] **OR/OF**	✓ R (1) ✓ R (1)

	$NKL = 180^\circ - K_4$ [adj. suppl.] $\therefore NKL = 180^\circ - NML$ [proved] $\therefore$ KLMN is a cyclic quad [opp.$\angle$s supplementary]	✓ R (1)
11.2.2	In $\Delta LKN \parallel\!\mid \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / $\angle$*e in dieselfde sirkel segm*] $\hat{L}_1 = \hat{M}_2$ [$\angle$s in the same seg / $\angle$*e in dieselfde sirkel segm*] $= \hat{K}_2$ [alt $\angle$s; / *verw*$\angle$*e;* MN||KS] $N\hat{K}L = M\hat{S}K$ [$\angle$s of Δ / $\angle$*e van* Δ] $\Delta LKN \parallel\!\mid \Delta KSM$ **OR/OF** In $\Delta LKN \parallel\!\mid \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / $\angle$*e in dieselfde sirkel segm*] $N\hat{K}L = \hat{M}_1$ [ext $\angle$ of cyclic quad/*buite*$\angle$ *van koordevh*] $= \hat{S}_2$ [corresp $\angle$s/*ooreenk* $\angle$*e*; KS || NM] $\Delta LKN \parallel\!\mid \Delta KSM$ [$\angle,\angle,\angle$] **OR/OF** In $\Delta LKN \parallel\!\mid \Delta KSM$: $\hat{N}_3 = \hat{M}_3$ [$\angle$s in the same seg / $\angle$*e in dieselfde sirkel segm*] $\hat{K}_4 + N\hat{K}L = \hat{S}_1 + \hat{S}_2$ [$\angle$s on straight line/$\angle$*e op reguitlyn*] $\therefore N\hat{K}L = \hat{S}_2$ [$\hat{K}_4 = \hat{S}_1$] $\Delta LKN \parallel\!\mid \Delta KSM$ [$\angle,\angle,\angle$]	✓ S ✓ R ✓ S ✓ S/R ✓ S (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5) ✓ S ✓ R ✓ S/R ✓ S ✓ R (5)
11.2.3	$\frac{LK}{KS} = \frac{KN}{SM}$ [$\Delta LKN \parallel\!\mid \Delta KSM$] $\therefore \frac{12}{KS} = \frac{4}{3}$ $KS = 9$ units	✓ S ✓ R ✓ substitution ✓ answer (4)
11.2.4	4SM = 3KN $SM = \frac{3(8)}{4}$ SM = 6 $\frac{LT}{NL} = \frac{LS}{ML}$ [line || one side of Δ / *lyn || een sy v* Δ] $\frac{LT}{16} = \frac{13}{19}$ $LT = \frac{208}{19} = 10{,}95$	✓ SM = 6 ✓ S ✓ R ✓ answer (4)
		[23]

TOTAL/*TOTAAL*: 150