basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P2

FEBRUARY/MARCH 2016

MARKS: 150

TIME: 3 hours

This question paper consists of 14 pages, 1 information sheet and a 25-page answer book.

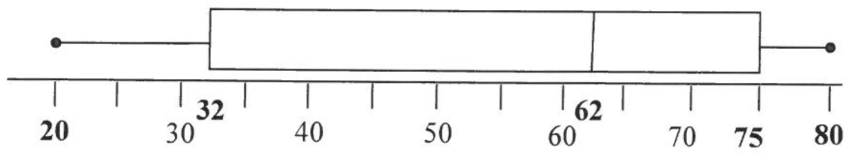
INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the SPECIAL ANSWER BOOK provided.
3. Clearly show ALL calculations, diagrams, graphs et cetera that you used to determine the answers.
4. Answers only will NOT necessarily be awarded full marks.
5. If necessary, round off answers to TWO decimal places, unless stated otherwise.
6. Diagrams are NOT necessarily drawn to scale.
7. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
8. An INFORMATION SHEET with formulae is included at the end of the question paper.
9. Write neatly and legibly.

QUESTION 1

The box and whisker diagram below shows the marks (out of 80) obtained in a History test by a class of nine learners.



1.1 Comment on the skewness of the data. (1)

1.2 Write down the range of the marks obtained. (2)

1.3 If the learners had to obtain 32 marks to pass the test, estimate the percentage of the class that failed the test. (2)

1.4 In ascending order, the second mark is 28, the third mark 36 and the sixth mark 69. The seventh and eighth marks are the same. The average mark for this test is 54.

	28	36			69			

Fill in the marks of the remaining learners in ascending order. (6)

[11]

QUESTION 2

A company recorded the number of messages sent by e-mail over a period of 60 working days. The data is shown in the table below.

NUMBER OF MESSAGES	NUMBER OF DAYS
$10 < x \leq 20$	2
$20 < x \leq 30$	8
$30 < x \leq 40$	5
$40 < x \leq 50$	10
$50 < x \leq 60$	12
$60 < x \leq 70$	18
$70 < x \leq 80$	3
$80 < x \leq 90$	2

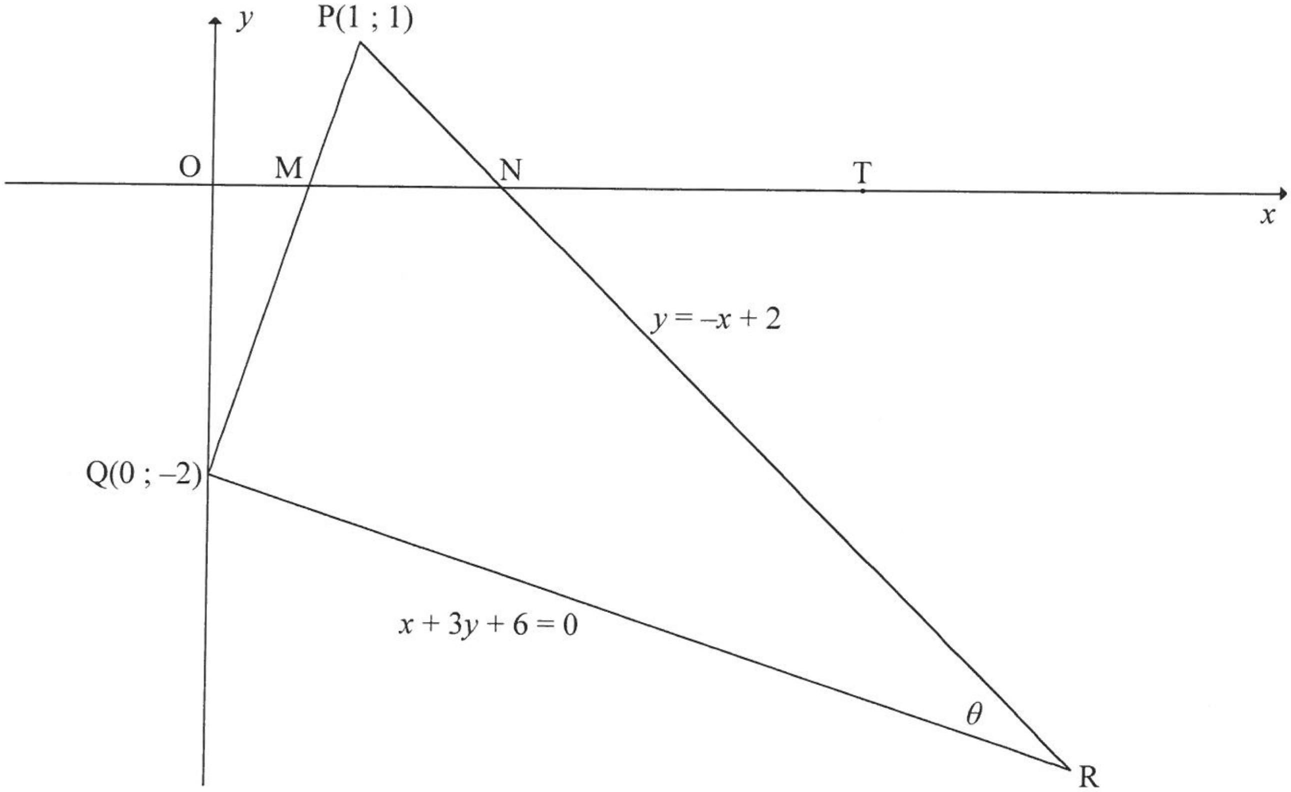
2.1 Estimate the mean number of messages sent per day, rounded off to TWO decimal places. (3)

2.2 Draw a cumulative frequency graph (ogive) of the data on the grid provided in the ANSWER BOOK. (4)

2.3 Hence, estimate the number of days on which 65 or more messages were sent. (2)
[9]

QUESTION 3

In the diagram below, P(1 ; 1), Q(0 ; –2) and R are the vertices of a triangle and $P\hat{R}Q = \theta$. The x-intercepts of PQ and PR are M and N respectively. The equations of the sides PR and QR are $y = -x + 2$ and $x + 3y + 6 = 0$ respectively. T is a point on the x-axis, as shown.



3.1 Determine the gradient of QP. (2)

3.2 Prove that $P\hat{Q}R = 90°$. (2)

3.3 Determine the coordinates of R. (3)

3.4 Calculate the length of PR. Leave your answer in surd form. (2)

3.5 Determine the equation of a circle passing through P, Q and R in the form $(x-a)^2 + (y-b)^2 = r^2$. (6)

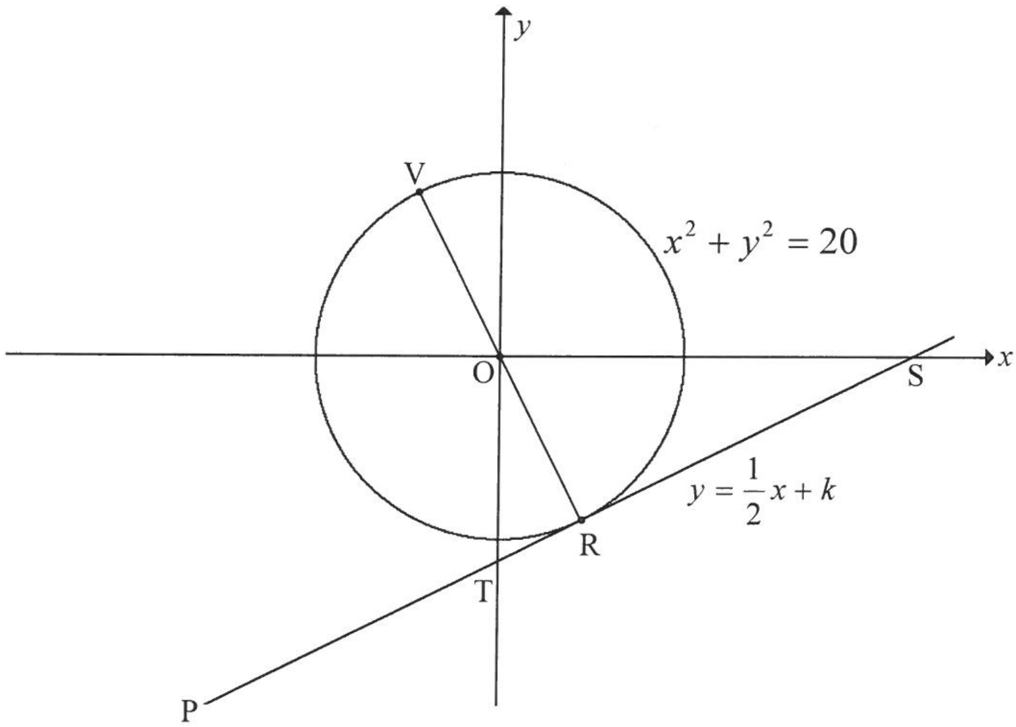
3.6 Determine the equation of a tangent to the circle passing through P, Q and R at point P in the form $y = mx + c$. (3)

3.7 Calculate the size of θ. (5)

[23]

QUESTION 4

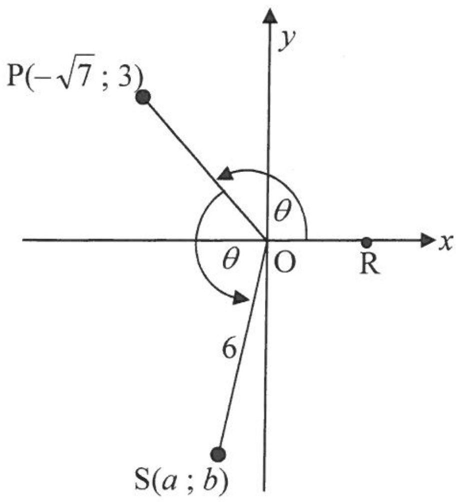
In the diagram below, the equation of the circle with centre O is $x^2 + y^2 = 20$. The tangent PRS to the circle at R has the equation $y = \frac{1}{2}x + k$. PRS cuts the y-axis at T and the x-axis at S.



4.1	Determine, giving reasons, the equation of OR in the form $y = mx + c$.	(3)
4.2	Determine the coordinates of R.	(4)
4.3	Determine the area of ΔOTS, given that R(2 ; –4).	(6)
4.4	Calculate the length of VT.	(4)
		[17]

QUESTION 5

5.1 P$(-\sqrt{7}\,;3)$ and S$(a\,;b)$ are points on the Cartesian plane, as shown in the diagram below. $\hat{POR} = \hat{POS} = \theta$ and OS = 6.



Determine, WITHOUT using a calculator, the value of:

5.1.1 $\tan\theta$ (1)

5.1.2 $\sin(-\theta)$ (3)

5.1.3 a (4)

5.2 5.2.1 Simplify $\dfrac{4\sin x\cos x}{2\sin^2 x-1}$ to a single trigonometric ratio. (3)

5.2.2 Hence, calculate the value of $\dfrac{4\sin 15°\cos 15°}{2\sin^2 15°-1}$ WITHOUT using a calculator. (Leave your answer in simplest surd form.) (2)

[13]

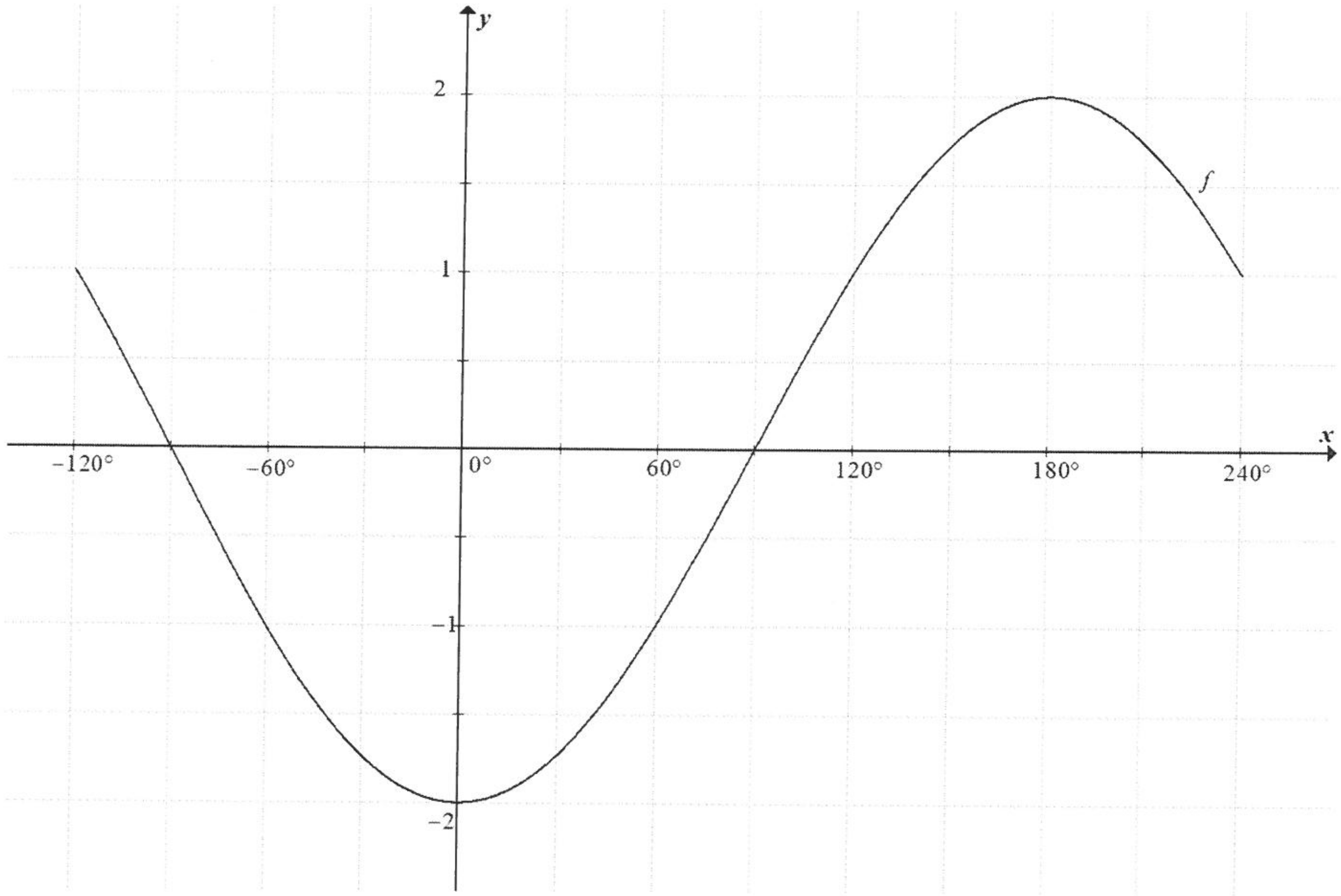
QUESTION 6

Given the equation: $\sin(x + 60°) + 2\cos x = 0$

6.1 Show that the equation can be rewritten as $\tan x = -4 - \sqrt{3}$. (4)

6.2 Determine the solutions of the equation $\sin(x + 60°) + 2\cos x = 0$ in the interval $-180° \leq x \leq 180°$. (3)

6.3 In the diagram below, the graph of $f(x) = -2\cos x$ is drawn for $-120° \leq x \leq 240°$.



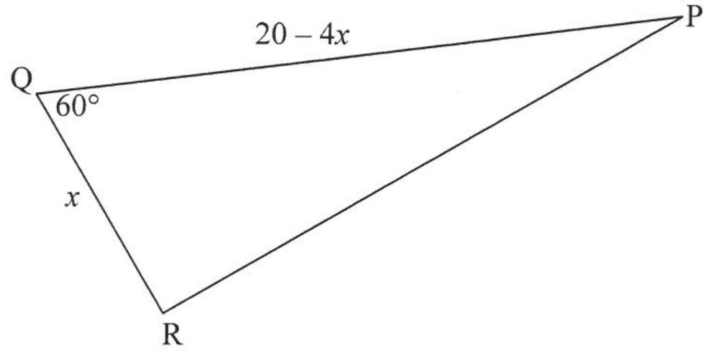
6.3.1 Draw the graph of $g(x) = \sin(x + 60°)$ for $-120° \leq x \leq 240°$ on the grid provided in the ANSWER BOOK. (3)

6.3.2 Determine the values of x in the interval $-120° \leq x \leq 240°$ for which $\sin(x + 60°) + 2\cos x > 0$. (3)

[13]

QUESTION 7

7.1 In the diagram below, ΔPQR is drawn with $PQ = 20 - 4x$, $RQ = x$ and $\hat{Q} = 60°$.

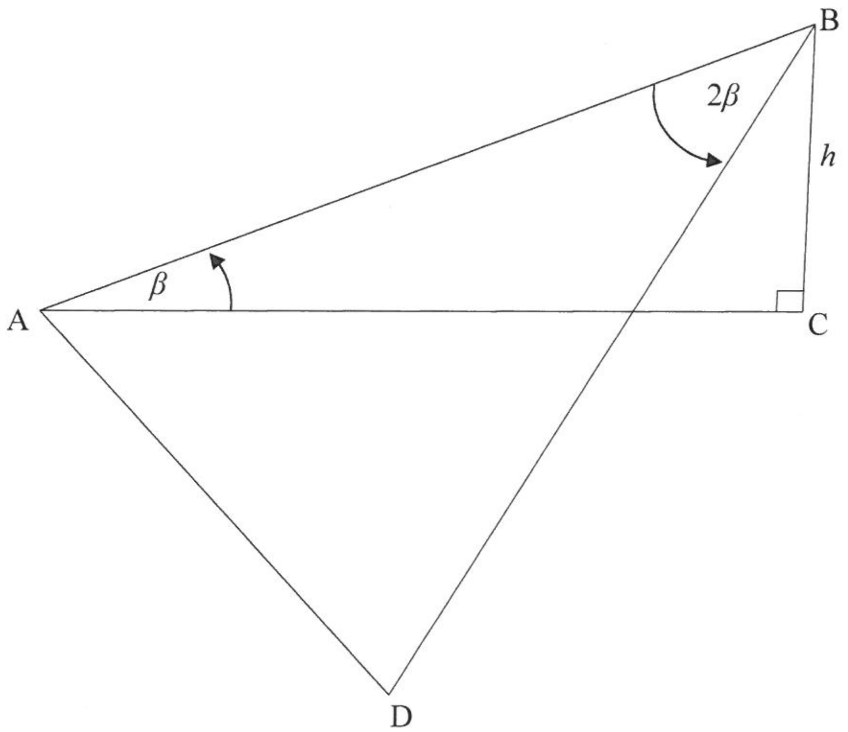


7.1.1 Show that the area of $\Delta PQR = 5\sqrt{3}x - \sqrt{3}x^2$. (2)

7.1.2 Determine the value of x for which the area of ΔPQR will be a maximum. (3)

7.1.3 Calculate the length of PR if the area of ΔPQR is a maximum. (3)

7.2 In the diagram below, BC is a pole anchored by two cables at A and D. A, D and C are in the same horizontal plane. The height of the pole is h and the angle of elevation from A to the top of the pole, B, is β. $A\hat{B}D = 2\beta$ and $BA = BD$.



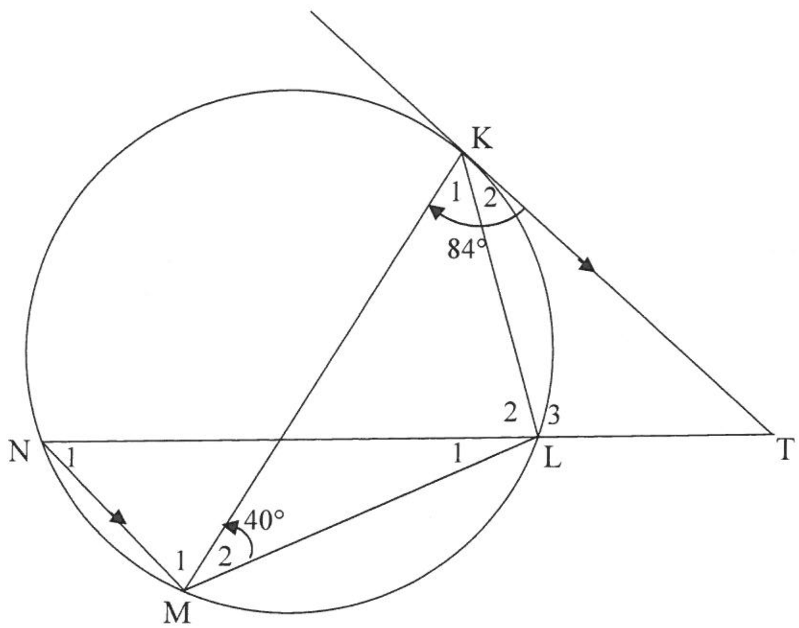
Determine the distance AD between the two anchors in terms of h. (7)

[15]

Give reasons for ALL statements in QUESTIONS 8, 9 and 10.

QUESTION 8

8.1 In the diagram below, tangent KT to the circle at K is parallel to the chord NM. NT cuts the circle at L. ΔKML is drawn. $\hat{M}_2 = 40°$ and $M\hat{K}T = 84°$.



Determine, giving reasons, the size of:

8.1.1 $\hat{K}_2$ (2)

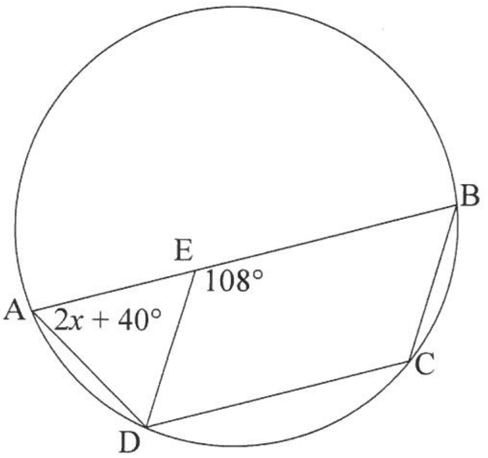
8.1.2 $\hat{N}_1$ (3)

8.1.3 $\hat{T}$ (2)

8.1.4 $\hat{L}_2$ (2)

8.1.5 $\hat{L}_1$ (1)

8.2 In the diagram below, AB and DC are chords of a circle. E is a point on AB such that BCDE is a parallelogram. $D\hat{E}B = 108°$ and $D\hat{A}E = 2x + 40°$.

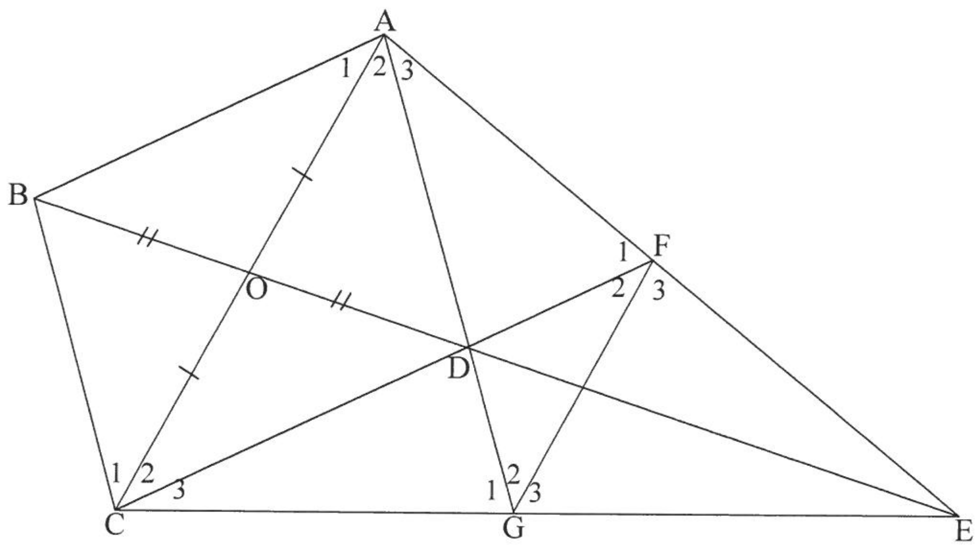


Calculate, giving reasons, the value of x. (5)

[15]

QUESTION 9

In the diagram below, EO bisects side AC of ΔACE. EDO is produced to B such that BO = OD. AD and CD produced meet EC and EA at G and F respectively.



9.1 Give a reason why ABCD is a parallelogram. (1)

9.2 Write down, with reasons, TWO ratios each equal to $\frac{ED}{DB}$. (4)

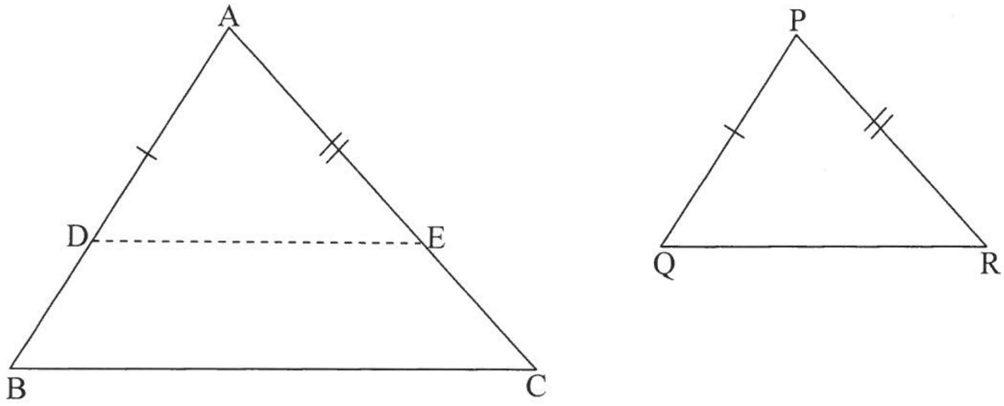
9.3 Prove that $\hat{A}_1 = \hat{F}_2$. (5)

9.4 It is further given that ABCD is a rhombus. Prove that ACGF is a cyclic quadrilateral. (3)

[13]

QUESTION 10

10.1 In the diagram below, ΔABC and ΔPQR are given with $\hat{A} = \hat{P}$, $\hat{B} = \hat{Q}$ and $\hat{C} = \hat{R}$.



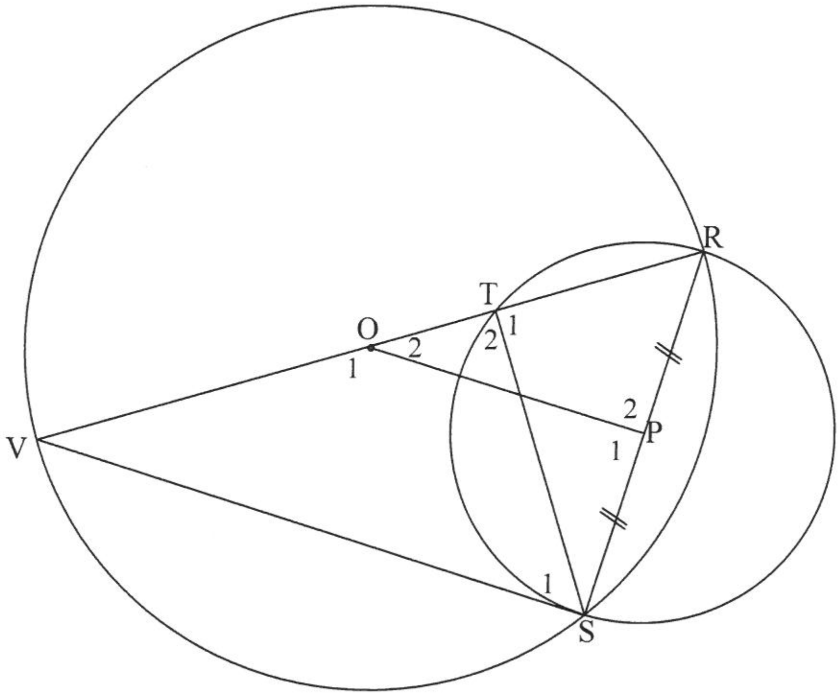
DE is drawn such that $AD = PQ$ and $AE = PR$.

10.1.1 Prove that $\Delta ADE \equiv \Delta PQR$. (2)

10.1.2 Prove that $DE \parallel BC$. (3)

10.1.3 Hence, prove that $\dfrac{AB}{PQ} = \dfrac{AC}{PR}$. (2)

10.2 In the diagram below, VR is a diameter of a circle with centre O. S is any point on the circumference. P is the midpoint of RS. The circle with RS as diameter cuts VR at T. ST, OP and SV are drawn.



10.2.1	Why is $OP \perp PS$?	(1)
10.2.2	Prove that $\Delta ROP \mid\mid\mid \Delta RVS$.	(4)
10.2.3	Prove that $\Delta RVS \mid\mid\mid \Delta RST$.	(3)
10.2.4	Prove that $ST^2 = VT \cdot TR$.	(6)
		[21]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$T_n = a + (n-1)d$ $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $S_n = \frac{a\left(r^n - 1\right)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\mathrm{M}\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$

$$(x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$$a^2 = b^2 + c^2 - 2bc.\cos A$$

$$area\,\Delta ABC = \frac{1}{2}ab.\sin C$$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n}$

$\mathrm{P(A)} = \frac{n(\mathrm{A})}{n(\mathrm{S})}$ P(A or B) = P(A) + P(B) – P(A and B)

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$